Exercise 12

Let L_n denote the left-endpoint sum using n subintervals and let R_n denote the corresponding right-endpoint sum. In the following exercises, compute the indicated left and right sums for the given functions on the indicated interval.

$$L_4 \text{ for } f(x) = \frac{1}{x-1} \text{ on } [2,3]$$

Solution

Since we're using the left-endpoint sum with $n = 4$ to approximate the integral of $f(x)$ from 2 to 3, the sum is taken from 0 to 3 rather than 1 to 4.

$$\int_2^3 f(x)\, dx \approx \sum_{i=0}^{3} f(x_i)\Delta x = \sum_{i=0}^{3} \frac{1}{x_i - 1}\Delta x$$

$$= \sum_{i=0}^{3} \frac{1}{(2 + i\Delta x) - 1}\Delta x$$

$$= \sum_{i=0}^{3} \frac{1}{1 + i\Delta x}\Delta x$$

$$= \sum_{i=0}^{3} \frac{1}{1 + i\left(\frac{3-2}{4}\right)}\left(\frac{3-2}{4}\right)$$

$$= \sum_{i=0}^{3} \frac{1}{1 + i\left(\frac{1}{4}\right)}\left(\frac{1}{4}\right)$$

$$= \sum_{i=0}^{3} \frac{1}{4 + i}$$

$$= \frac{1}{4+0} + \frac{1}{4+1} + \frac{1}{4+2} + \frac{1}{4+3}$$

$$= \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7}$$

$$= \frac{319}{420}$$

$$\approx 0.760$$